MATH 1650: SECTION 6.6: APPLICATIONS OF EXPONENTIAL FUNCTIONS

Compounded Interest:
If an initial principal P is invested at an annual rate r and the interest is compounded n times per year, the amount in the account after t years, $A(t)$ is given by

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$$

Continuously Compounded Interest:
If an initial principal P is invested at an annual rate r and the interest is compounded continuously, the amount in the account after t years, $A(t)$ is given by

$$A(t) = Pe^{rt}$$

Uninhibited Growth:
If a population increases according to The Law of Uninhibited Growth, the number of organisms at time t, $N(t)$ is given by the formula

$$N(t) = N_0 e^{kt},$$

where $N(0) = N_0$ (read 'N nought') is the initial number of organisms and $k > 0$ is growth rate.

Radioactive Decay:
The amount of a radioactive element at time t, $A(t)$ is given by the formula

$$A(t) = A_0 e^{kt},$$

where $A(0) = A_0$ is the initial amount of the element and $k < 0$ is the decay rate.

Newton's Law of Cooling (Warming):
The temperature of an object at time t, $T(t)$ is given by the formula

$$T(t) = T_a + (T_0 - T_a)\, e^{-kt},$$

where $T(0) = T_0$ is the initial temperature of the object, T_a is the ambient temperature[1] and $k > 0$ is a constant.

Logistic Growth:
If a population behaves according to the assumptions of logistic growth, the number of organisms at time t, $N(t)$ is given by

$$N(t) = \frac{L}{1 + Ce^{-kLt}},$$

where $N(0) = N_0$ is the initial population, L is the limiting population,[2] and C is a measure of how much room there is to grow given by

$$C = \frac{L}{N_0} - 1.$$

and $k > 0$ is a constant.

[1] That is, the temperature of the surroundings.
[2] That is, as $t \to \infty$, $N(t) \to L$

SAMPLE PROBLEMS

1. An account offers 5% interest, compounded monthly.

   How much will be in the account in 30 years if \$450 is invested today?

2. An account offers 2.25% interest, compounded continuously.

   How much should Sally invest today if she wants \$5000 in 10 years?

3. Skippy invests in an account offers 3.25% interest.

   (a) Find the doubling time if the interest is compounded *monthly*.

   (b) Find the doubling time if the interest is compounded *continuously*.

4. *Eludium Phosdax*, the so-called 'shaving cream atom' decays according to the formula $A(t) = A_0 e^{kt}$.

   (a) If the half-life of this element is 25 minutes, find and interpret the decay constant, k.
   HINT: The half-life of the material is how long it takes for half of the material to decay.
   A_0 is the initial amount of the material, so after the half-life, there would be $\frac{1}{2}A_0$ of the material left.

   (b) How long does it take for 90% of this material to decay?
   Find an exact answer, then find an approximate the answer, rounded to the nearest minute.
   HINT: If 90 % decays how much is left?

5. Newton's Law of Cooling states that the temperature T of an object at time t is given by:

$$T(t) = T_a + (T_0 - T_a)\,e^{-kt},$$

where $T(0) = T_0$ is the initial temperature of the object, and T_a is the ambient temperature.[3]

Suppose a piping hot cup of coffee (at 180°F) is served in a room that is 72°.

After 10 minutes, the coffee is 155°F.

(a) If the coffee cools according to Newton's Law of Cooling, find a formula for the temperature of the coffee in degrees Fahrenheit, $T(t)$, as a function of the number of minutes it has been left to cool, t.

(b) How long does take for the coffee to cool to 120°F ?

(c) Find and interpret the horizontal asymptote to the graph of $y = T(t)$.

[3] That is, the temperature of the surroundings.

6. The population of Sasquatch, P, on Roskos Acres t years after 2003 is given by:

$$P(t) = \frac{150}{2 + 73e^{-0.5t}}, \quad t \geq 0$$

(a) Find and interpret $P(0)$.

(b) What is the population in 2007?

(c) Solve $P(t) = 50$ and interpret.

(d) Find and interpret the horizontal asymptote of the graph of $y = P(t)$.

HOMEWORK: Section 6.6: 1 - 37 every other odd.